Orbit Determination of 1951 Lick

Fengning Ding, Jason K. Liu, Patrick Rall
Summer Science Program 2011 at Westmont College

Abstract—During the summer of 2011, the asteroid 1951 Lick was observed on seven days using a 14" Meade telescope and the 24" Keck Telescope at Westmont College. From seventeen usable images, the right ascension and declination of the asteroid were measured using least-squares plate reductions. Because the asteroid had just completed a retrograde loop at the time of observation, the rate of change of the vector from the Earth to the asteroid was small and difficult to compute. Existing well-developed orbital determination methods such as Gauss-Gibbs's method and Laplace's method are very sensitive to this rate of change, and both methods failed to give accurate orbital parameters. By using Laplace's method to first give reasonably correct estimates and then using five iterations of differential corrections to refine the preliminary results, it was possible to compute the orbit of the asteroid. While one iteration of differential corrections is often sufficient to refine the result, the initial estimate required multiple iterations to compensate for the asteroid's position in its orbit. This method produced spectacular results; relative to an observation made on July 19, the ephemeris calculated from the orbital elements determined was within 0.112 seconds of the right ascension and 1.56 arcseconds of the declination of the observed coordinates of the asteroid, confirming the consistency of the orbital elements from this analysis with those of previously determined measurements. This research demonstrates the possibility of computing orbits and furnishing excellent orbital parameters even in adverse situations using the method of repeated differential corrections.

1 INTRODUCTION

DUE to the potential of some near-Earth asteroids to collide with Earth, astronomers are especially interested in the orbits of near-Earth asteroids. To ensure accurate monitoring of these hazardous objects, which often move in chaotic orbits perturbed by other planets, astronomers need to repeatedly determine the orbit of the asteroid with high precision. The orbit is characterized by six parameters corresponding to the six degrees of freedom of the initial conditions: the semimajor axis a, the eccentricity e, the inclination with respect to the plane of the Earth's orbit i, the longitude of the ascending node with respect to the vernal equinox Ω, the argument of the perihelion ω, and the time of the last perihelion passage T_p. These orbital parameters can be used to locate the asteroid in the sky or to generate the position of the asteroid in the future.

In July 2011, as part of the Summer Science Program at Westmont College, the orbit of the asteroid 1951 Lick was determined. A 14″ Meade telescope and the 24″ Keck Telescope at Westmont College were used to take seventeen images on seven different nights. These images were subsequently used to determine the asteroid's right ascension α and declination δ at specific times. The dataset of seventeen images is augmented by data collected by another team (Hyde, Shah, and Zhao) observing 1951 Lick.

Because the asteroid was leaving a retrograde loop, the equatorial unit vector $\hat{\rho}$ from the Earth to the asteroid changed very slowly, causing the classical Gauss-Gibbs orbit determination method, which is sensitive to $\frac{d\hat{\rho}}{dt}$, to give nonsensical values. Laplace's method, another classical method of orbit determination, also initially gave inaccurate results. However, since the orbital parameters calculated from Laplace's method were at least very approximately correct, five iterations of differential corrections gave accurate orbital elements for the asteroid. From the computed orbital elements, which are consistent with previously generated results, an ephemeris accurately matching the observed coordinates of the asteroid was produced.

Date	Telescope	Sync Star	Filter	# images
July 1	Meade 14″	Omitted due to bad quality		
July 3	Keck 24″	Arcturis	Clear	5
			Clear	5
		Omitted due to bad quality		
July 8	Meade 14″	Images are of wrong part of sky		
		Phecda	Clear	7
			Clear	7
			Clear	7
July 10	Meade 14″	Phecda	Clear	7
			Clear	7
			Clear	7
July 16	Meade 14″	Phecda	V-filter	5
			Clear	5
			Clear	5
			Clear	5
			Clear	5
July 19	Meade 14″	Phecda	Clear	7
			Clear	7
			Clear	7
			Clear	7
July 25	Meade 14″	Phecda	Clear	5
			V-filter	1

TABLE 1
List of image series taken in observations of 1951 Lick in 2011

One novel aspect of this work is the application of multiple iterations of differential corrections. To determine the orbit from most sets of observations, only one iteration is required to refine the results of either Gauss-Gibbs's method or Laplace's method. However, because of the adverse positions of the Earth and the asteroid in their orbits, one iteration of differential corrections was not sufficient to correct for the rough estimates yielded by Laplace's method. This extended orbital determination method could allow future astronomers to compute orbits of objects in similarly unfavorable circumstances.

2 METHODS

The images used for orbit determination were taken using two telescopes. After median-combining these images into series to reduce noise, a least-squares plate reduction (LSPR) was calculated using star coordinate data from the Naval Observatory Merged Astrometric Dataset (NOMAD) database. These reductions yielded the right ascension α and declination δ for the asteroid at the observation times.

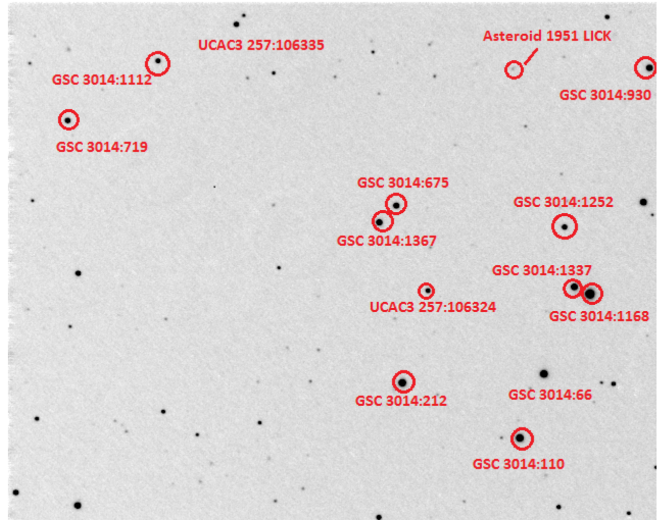

Fig. 1. An example of a combined series of images: Series 2 of the observation on July 10 using the Meade 14" telescope.

The orbital elements were determined using an implementation of Laplace's method of orbit determination with five iterations of differential corrections.

2.1 Observing and Image Processing

The asteroid was observed on seven nights: July 1, July 3, July 8, July 10, July 16, July 19, and July 25, 2011 from 04:00 to 06:00 Coordinated Universal Time (UTC). First, the apparent α and δ of the asteroid was obtained from the NASA Jet Propulsion Laboratory's HORIZONS ephemeris computation service. TheSkyX software by Software Bisque was used to generate a star chart of the region of the sky in which the asteroid would be located. After synchronizing and focusing the telescope on a nearby star, three or four series of five or seven images were taken with an exposure time of 45 seconds per image. For details see Table 1. For an example of a processed image see Fig. 1.

When retrieving the approximate α and δ from JPL HORIZONS, data was collected for the apparent α and δ, which compensated for various systematic errors such as atmospheric distortion. However, the telescope already corrects for these effects, and so the asteroid was always located near the edge of the image.

This error was discovered on July 19, and was eliminated in subsequent observations.

To remove noise and 'hot pixels' (pixels that had been overexposed by a cosmic ray), the images for each series were flat-field corrected, manually aligned, and median combined using the MaxIm DL software by Diffraction Limited to form three processed images per observation. The asteroid was identified on the three processed images of each observation by aligning and 'blinking' (changing images over a short time period) the three images.

2.2 Least-Squares Plate Reduction

Using a Python program written by the authors that implements least-squares plate reduction (LSPR), the equatorial coordinates of the asteroid in each image were calculated. LSPR is a process to find the mapping from the x,y plate coordinates of bright reference stars (which are obtained from Maxim DL) to the equatorial coordinates of each star tabulated in the NOMAD database. By applying this map to the x,y coordinates of the asteroid, the equatorial coordinates of the asteroid were obtained.

Before LSPR was performed, the effect of altitude on the stars' angle of refraction was accounted for. The time of observation given by each image's file information header and the difference between the observatory's clock and the US Naval Observatory's clock were used to calculate the local sidereal time. This, in turn, was used to convert the equatorial coordinates of each reference star from NOMAD into theoretical values for local coordinates of altitude and azimuth. Using an index of refraction of $n-1 = 58.2''$ for the atmosphere,[1] the computed altitude of each star was adjusted to account for the resulting offset. The adjusted altitude and azimuth were then converted back to theoretical apparent equatorial coordinates.

To find the mapping between the plate x,y coordinates and the equatorial α and δ coordinates by LSPR, a least-squares linear fit that mapped the x,y plate coordinates of each star to its theoretical apparent equatorial coordinates was computed. Using this mapping, the apparent equatorial coordinates of the center of the image were obtained to calculate the standard ξ,η coordinates for each reference star.[2] Because the standard coordinates are truly linear functions of the plate coordinates, a more accurate least-squares fit was computed that maps the plate coordinates to the standard coordinates. Using this linear transformation, the ξ,η coordinates of the asteroid were obtained, from which the apparent α,δ of the asteroid were calculated. Finally, the apparent equatorial coordinates were transformed back to equatorial coordinates using the same index of refraction of the atmosphere.

To obtain an estimate of the uncertainty of the α and δ of the asteroid, the deviation of the predicted equatorial coordinates of each star from the catalogued coordinates were computed assuming two degrees of freedom. This residual was possibly due to defects in the CCD chip or telescope mirror, aberration of starlight, inaccurate tabulations of reference star coordinates, or uncertainties in the centroid computations.

2.3 Orbit Determination using Laplace's Method

From the measured α and δ of the asteroid, the classical orbital elements of the asteroid were determined. Because the asteroid had recently exited from a retrograde loop, the asteroid's position relative to Earth as a function of time was approximately a straight line, resulting in small values for the change in velocity of the unit range vector $\hat{\rho}$, $\frac{d^2\hat{\rho}}{d\tau^2}$.[3] The Gauss-Gibbs method of orbit determination,[4] which is sensitive to values of $\frac{d^2\hat{\rho}}{d\tau^2}$, produced unrealistic results, and so Laplace's method was used.

Five observations were used to obtain $\frac{d\hat{\rho}}{d\tau}$ and $\frac{d^2\hat{\rho}}{d\tau^2}$ for the middle observation. The five unit equatorial vectors are given by $\hat{\rho}_1$, $\hat{\rho}_2$, $\hat{\rho}_3$, $\hat{\rho}_4$ and $\hat{\rho}_5$. The middle range vectors give the position of the asteroid on July 8 and were used to accurately estimate $\frac{d\hat{\rho}}{d\tau}$. The second derivative

1. Value as in [2]

2. See [2]

3. τ is the modified time given by $\tau = kt$, for ordinary time t and the Gaussian gravitation constant $k = 0.017...$ All time derivatives in this section are with respect to modified time.

4. Described in [1]

of $\hat{\rho}$ at the middle observation was approximated using $\hat{\rho}_1$, $\hat{\rho}_3$, and $\hat{\rho}_5$, the range vectors of the asteroid on June 27, July 8, and July 25 respectively.[5]

This time span was chosen to be large enough for the first derivative to change sufficiently.

To compute the first derivative, a second-order Taylor expansion was used:

$$\begin{aligned}\hat{\rho}_4 &= \hat{\rho}_3 + \hat{\rho}_3'(\tau_4 - \tau_3) + \frac{1}{2}\hat{\rho}_3''(\tau_4 - \tau_3)^2 \\ \hat{\rho}_2 &= \hat{\rho}_3 + \hat{\rho}_3'(\tau_2 - \tau_3) + \frac{1}{2}\hat{\rho}_3''(\tau_2 - \tau_3)^2\end{aligned}$$

which was solved for $\hat{\rho_3}'$. The second derivative was calculated similarly but with $\hat{\rho}_1$ and $\hat{\rho}_5$ instead of $\hat{\rho}_2$ and $\hat{\rho}_4$. From $\frac{d\hat{\rho}}{d\tau}$ and $\frac{d^2\hat{\rho}}{d\tau^2}$, the standard Laplace method was used to generate the position and velocity vectors of the asteroid.[6] This calculation was done using a script written in Python.

From the preliminary position and velocity, an ephemeris was generated for the times of each of the observations, and the differences from the measured values $\Delta\alpha$ and $\Delta\delta$ were calculated. The difference between the computed and measured values of α and δ is caused by a small error in the position and velocity according to the equations below:

$$\begin{aligned}\Delta\alpha &= \frac{\partial\alpha}{\partial r_x}\Delta r_x + \frac{\partial\alpha}{\partial r_y}\Delta r_y + \frac{\partial\alpha}{\partial r_z}\Delta r_z + \\ &\quad \frac{\partial\alpha}{\partial v_x}\Delta v_x + \frac{\partial\alpha}{\partial v_y}\Delta v_y + \frac{\partial\alpha}{\partial v_z}\Delta v_z \\ \Delta\delta &= \frac{\partial\delta}{\partial r_x}\Delta r_x + \frac{\partial\delta}{\partial r_y}\Delta r_y + \frac{\partial\delta}{\partial r_z}\Delta r_z + \\ &\quad \frac{\partial\delta}{\partial v_x}\Delta v_x + \frac{\partial\delta}{\partial v_y}\Delta v_y + \frac{\partial\delta}{\partial v_z}\Delta v_z\end{aligned}$$

Using six observations of the asteroid, least-squares corrections Δr_x, Δr_y, Δr_z, Δv_x, Δv_y, and Δv_z for the position and velocity vectors were found. Due to inaccurate preliminary vectors, these corrected positions and velocities were still not accurate since the differential corrections described above only correct for small inaccuracies. This process was iterated five times until the computed orbital elements converged, with the last position and velocity vectors giving new corrected position and velocity vectors according to the above procedure. As Table 2 shows, Laplace's method gives inaccurate results for ω and T_P. With one iteration of corrections, ω and T_P are much more accurate, but the other orbital elements are over-corrected. Each subsequent iteration improves the value of all orbital elements.

No. of Corrections	a [AU]	e	i [°]
Laplace's Method	1.376	0.0699	39.447
One iteration	1.342	0.0576	38.968
Two iterations	1.384	0.0592	39.078
Three iterations	1.389	0.0604	39.087
Four iterations	1.389	0.0604	39.088
Five iterations	1.389	0.0605	39.088
HORIZONS Value	1.390536	0.0616082	39.08962
No. of Corrections	**Ω [°]**	**ω [°]**	**T_P [JD]**
Laplace's Method	129.761	344.711	2455602.859
One iteration	131.535	163.598	2455298.593
Two iterations	130.852	142.718	2455243.991
Three iterations	130.777	140.797	2455238.203
Four iterations	130.773	140.700	2455237.901
Five iterations	130.772	140.695	2455237.885
HORIZONS Value	130.769445	140.4418	2455236.658

TABLE 2
1951 Lick orbital element outputs of Laplace's method, iterations of differential correction, and JPL HORIZONS

To compute the uncertainties for the orbital elements, a normal distribution according to the standard deviation computed from the LSPR for the α and δ of the asteroid was assumed. A Monte-Carlo method with sample size of five hundred was used to compute the orbital elements using the procedure specified above for a set of asteroid coordinates randomly sampled from the Gaussian distribution. The standard deviation of the computed orbital elements was reported as the uncertainty.

2.4 Photometry

On July 16 and July 25, 2011, a V-filter image of the asteroid was taken to measure the V-magnitude of the asteroid. To determine the apparent magnitude of the asteroid, images were taken with a V-filter in the telescope, instead of the clear filter that was used for astrometry.

5. The June 27 data were obtained by Hyde, Shah, and Zhao
6. See [1]

Asteroid V-mag	Observation	Sync Star V-mag
16.861	GSC 2531:1730	12.690
17.122	GSC 2529:1728	12.590

TABLE 3
V-magnitude of 1951 Lick

a	e	i
1.3894 ± 0.0094 AU	0.06053 ± 0.0024	$39.089 \pm 0.019°$
Ω	ω	T_P
$130.7706 \pm 0.1415°$	$140.65 \pm 3.71°$	2455237.7 ± 11.3 JD

TABLE 4
Measured orbital elements of 1951 Lick

Quantity	Measurement	HORIZONS Value	Difference
a	1.3894 ± 0.0094 AU	1.390536 AU	0.0011 AU
e	0.06053 ± 0.0024	0.0616082	0.0011
i	$39.089 \pm 0.019°$	39.08962°	0.001°
Ω	$130.7706 \pm 0.1415°$	130.769445°	0.0012°
ω	$140.65 \pm 3.71°$	140.4418°	0.21°
T_P	2455237.7 ± 11.3 JD	2455236.658 JD	1.04 JD

TABLE 5
Measured orbital elements in comparison with JPL HORIZONS orbital elements of 1951 Lick

After taking a series of images with the V-filter, V-magnitudes of surrounding stars were obtained from the NOMAD database. From the V-magnitudes of the reference stars, MaximDL computed the V-magnitude of the asteroid. See Table 3.

3 Results

The result of the orbit determination along with the uncertainty are shown in Table 4.

The eccentricity of the orbit of the asteroid is similar to that of Mars with $e = 0.093$ [3], and the point of perihelion is hard to distinguish from other points, so the values for ω and T_p are more difficult to determine than the other orbital elements.

4 Conclusion

JPL HORIZONS provides values for the orbital elements, which serve as a good comparison to computed results. The differences between the orbital elements that were determined in the experimental procedure and those given by JPL are shown in Table 5. The discrepancies

Date	Observed α	HORIZONS α	Computed α
	Observed δ	HORIZONS δ	Computed δ
Jul 19	12h 24m 7.855s	12h 24m 3.18s	12h 24m 7.667s
	35° 5′ 27.33″	35° 5′ 32.0″	35° 5′ 28.89″

TABLE 6
Comparison of JPL HORIZONS ephemeris of 1951 Lick and ephemeris from determined orbital elements with observation data

are significantly less than the statistical errors. An ephemeris for July 19 generated from determined orbital elements was also calculated and compared to observations, shown in Table 6. Since orbital elements are constantly changing due to perturbations from planets, it is irrelevant to compare ephemerides generaged from these orbital elements with those generated from orbital elements from HORIZONS, because the orbital elements of this study are fitted to the observations and not the orbit at any precise epoch. However, the accuracy of this orbit determination is of theoretical interest, because it demonstrates that accurate orbits can be determined even if Gauss-Gibbs's method diverges and Laplace's method gives orbital elements that are not within reasonable accuracy.

Acknowledgments

The authors would like to thank Dr. Michael Faison, Martin Mason, and Mary Masterman, Dougal Sutherland, Becky Raph, and Sean Mattingly for their help debugging Python scripts, assistance with observations, and teaching us the methodology used in this project. The authors are also grateful towards Jeremy Hyde, Pathik Shah, and Ming Zhao for sharing their data of 1951 Lick.

References

[1] Danby, J. M. A. (1962). *Fundamentals of Celestial Mechanics* New York: Macmillan, 1962.
[2] Tatum, J. B. (2002). *Celestial Mechanics* Obtained from M. Mason in 2011
[3] NASA Jet Propulsion Laboratory HORIZONS System. Retrieved from http://ssd.jpl.nasa.gov/?horizons.